ClarityEd

Physics: Mechanics

Equation Summary

Newton's Second Law: Force, Mass, Acceleration: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$

Newton's Third Law: $\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$

Center of Mass: $\vec{\mathbf{R}}_{\text{cm}} = \frac{1}{m^{\text{total}}}\sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i \rightarrow \frac{1}{m^{\text{total}}}\int_{\text{body}} dm\,\vec{\mathbf{r}}$;

Velocity of Center of Mass: $\vec{\mathbf{V}}_{\text{cm}} = \frac{1}{m^{\text{total}}}\sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i \rightarrow \frac{1}{m^{\text{total}}}\int_{\text{body}} dm\,\vec{\mathbf{v}}$

Momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, $\vec{\mathbf{p}}^{\text{sys}} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i$

Newton's Second Law $\vec{\mathbf{F}}^{ext} = \frac{d\vec{\mathbf{p}}^{sys}}{dt}$

Impulse: $\vec{\mathbf{I}} \equiv \int_{t=0}^{t=t_f} \vec{\mathbf{F}}(t)dt = \Delta\vec{\mathbf{p}}$

Kinetic Energy: $K = \frac{1}{2}mv^2$; $\Delta K \equiv \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$

Work- Kinetic Energy: $W = \int_A^B \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ $\quad W = \Delta K$

Potential Energy: $\Delta U \equiv U(B) - U(A) \equiv -W_{\text{c}} = -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{\text{gravity}}(\text{r}) = -\frac{Gm_1m_2}{r}$ with $U_{\text{gravity}}(\text{r}_0 = \infty) = 0$.

Springs: $U_{\text{spring}}(x) = \frac{1}{2}kx^2$ with $U_{\text{spring}}(x = 0) = 0$.

Work- Mechanical Energy: $W_{\text{nc}} = \Delta K + \Delta U^{\text{total}} = \Delta E_{\text{mech}} = \left(K_f + U_f^{\text{total}}\right) - \left(K_0 + U_0^{\text{total}}\right)$

Moment of Inertia: $$I_P = \int_{\text{body}} dm(r_\perp)^2$$

Moment of inertia of uniform disk of mass M and radius R about axis passing through center of mass perpendicular to plane of disk: $(1/2)MR^2$

Moment of inertia of uniform disk of mass M and radius R about axis passing through center of mass parallel to plane of disk: $(1/4)MR^2$

Moment of inertia of uniform rod of mass M and length L about axis passing through center of mass perpendicular to rod: $(1/12)ML^2$

Parallel Axis Theorem: $I_P = md^2 + I_{cm}$

Torque about a point S: $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,F} \times \vec{\mathbf{F}}$

Angular Momentum (point particle) about a point S: $\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_S \times m\vec{\mathbf{v}}$

Angular Impulse: $$\int_{t_i}^{t_f} \vec{\boldsymbol{\tau}}_S^{\text{ext}}\, dt = \vec{\mathbf{L}}_{S,f} - \vec{\mathbf{L}}_{S,i}$$

Fixed Axis Rotation (about z-axis):

Angular Velocity: $\vec{\boldsymbol{\omega}} = \omega_z \hat{\mathbf{k}}$

Angular Acceleration: $\vec{\boldsymbol{\alpha}} = \alpha_z \hat{\mathbf{k}}$

Angular Momentum for fixed axis rotation (symmetric body): $\vec{\mathbf{L}}_z = I_z \omega_z \hat{\mathbf{k}}$

Torque and Angular momentum about point S: $$\vec{\boldsymbol{\tau}}_S^{\text{ext}} = \frac{d\vec{\mathbf{L}}_S^{\text{sys}}}{dt}$$

Rotational Kinetic Energy about fixed point S: $$K_S^{\text{rot}} = \frac{1}{2} I_S \omega^2$$

Rotation and Translation:

Angular Momentum about a point S: $\vec{\mathbf{L}}_S = \vec{\mathbf{L}}_S^{orbital} + \vec{\mathbf{L}}_{cm}^{spin} = (\vec{\mathbf{r}}_{S,cm} \times m\vec{\mathbf{v}}_{cm}) + \vec{\mathbf{L}}_{cm}^{spin}$

Torque about a point: $$\vec{\boldsymbol{\tau}}_S = \frac{d\vec{\mathbf{L}}_S}{dt} \quad \text{(fixed point } S\text{)}, \quad \vec{\tau}_{cm} = \frac{d\vec{\mathbf{L}}_{cm}^{spin}}{dt} \quad \text{(center of mass)}$$

Kinetic Energy: $$K = K_{trans} + K_{rot} = \frac{1}{2} m_{total} v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$