The Celestial Sphere

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1 ■ THE GREEK TRADITION

Human beings have long looked up at the sky and pondered its mysteries. Evidence of the long struggle to understand its secrets may be seen in remnants of cultures around the world: the great Stonehenge monument in England, the structures and the writings of the Maya and Aztecs, and the medicine wheels of the Native Americans. However, our modern scientific view of the universe traces its beginnings to the ancient Greek tradition of natural philosophy. Pythagoras (ca. 550 B.C.) first demonstrated the fundamental relationship between numbers and nature through his study of musical intervals and through his investigation of the geometry of the right angle. The Greeks continued their study of the universe for hundreds of years using the natural language of mathematics employed by Pythagoras. The modern discipline of astronomy depends heavily on a mathematical formulation of its physical theories, following the process begun by the ancient Greeks.

In an initial investigation of the night sky, perhaps its most obvious feature to a careful observer is the fact that it is constantly changing. Not only do the stars move steadily from east to west during the course of a night, but different stars are visible in the evening sky, depending upon the season. Of course the Moon also changes, both in its position in the sky and in its phase. More subtle and more complex are the movements of the planets, or "wandering stars."

The Geocentric Universe

Plato (ca. 350 B.C.) suggested that to understand the motions of the heavens, one must first begin with a set of workable assumptions, or hypotheses. It seemed obvious that the stars of the night sky revolved about a fixed Earth and that the heavens ought to obey the purest possible form of motion. Plato therefore proposed that celestial bodies should move about Earth with a uniform (or constant) speed and follow a circular motion with Earth at the center of that motion. This concept of a **geocentric universe** was a natural consequence of the apparently unchanging relationship of the stars to one another in fixed constellations.

The Celestial Sphere

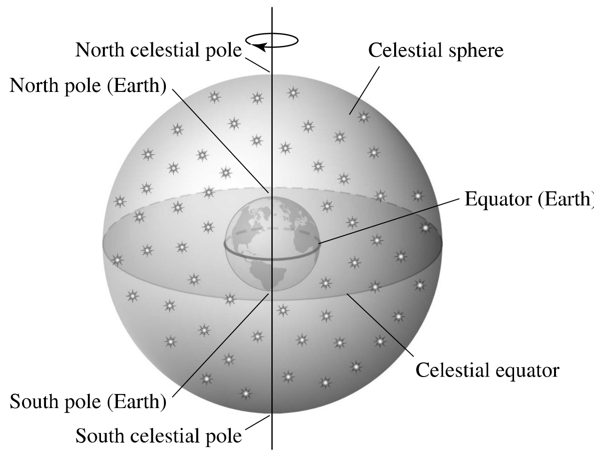


FIGURE 1 The celestial sphere. Earth is depicted in the center of the celestial sphere.

If the stars were simply attached to a **celestial sphere** that rotated about an axis passing through the North and South poles of Earth and intersecting the celestial sphere at the **north** and **south celestial poles**, respectively (Fig. 1), all of the stars' known motions could be described.

Retrograde Motion

The wandering stars posed a somewhat more difficult problem. A planet such as Mars moves slowly from west to east against the fixed background stars and then mysteriously reverses direction for a period of time before resuming its previous path (Fig. 2). Attempting to understand this backward, or **retrograde**, **motion** became the principal problem in astronomy for nearly 2000 years! Eudoxus of Cnidus, a student of Plato's and an exceptional mathematician, suggested that each of the wandering stars occupied its own sphere and that all the spheres were connected through axes oriented at different angles and rotating at various speeds. Although this theory of a complex system of spheres initially was marginally successful at explaining retrograde motion, predictions began to deviate significantly from the observations as more data were obtained.

Hipparchus (ca. 150 B.C.), perhaps the most notable of the Greek astronomers, proposed a system of circles to explain retrograde motion. By placing a planet on a small, rotating **epicycle** that in turn moved on a larger **deferent**, he was able to reproduce the behavior of the wandering stars. Furthermore, this system was able to explain the increased brightness of the planets during their retrograde phases as resulting from changes in their distances from Earth. Hipparchus also created the first catalog of the stars, developed a magnitude system for describing the brightness of stars that is still in use today, and contributed to the development of trigonometry.

During the next two hundred years, the model of planetary motion put forth by Hipparchus also proved increasingly unsatisfactory in explaining many of the details of the observations. Claudius Ptolemy (ca. A.D. 100) introduced refinements to the epicycle/deferent

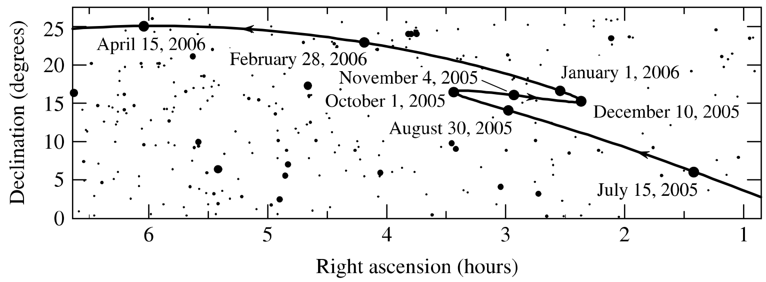


FIGURE 2 The retrograde motion of Mars in 2005. The general, long-term motion of the planet is eastward relative to the background stars. However, between October 1 and December 10, 2005, the planet's motion temporarily becomes westward (retrograde). (Of course the planet's short-term daily motion across the sky is always from east to west.) The coordinates of right ascension and declination are discussed in Fig. 13. Betelgeuse, the bright star in the constellation of Orion, is visible at $(\alpha, \delta) = (5^h55^m, +7^\circ 24')$, Aldebaran, in the constellation of Taurus, has coor-dinates $(4^h36^m, +16^\circ 31')$, and the Hyades and Pleiades star clusters (also in Taurus) are visible at $(4^h24^m, +15^\circ 45')$ and $(3^h44^m, +23^\circ 58')$, respectively.

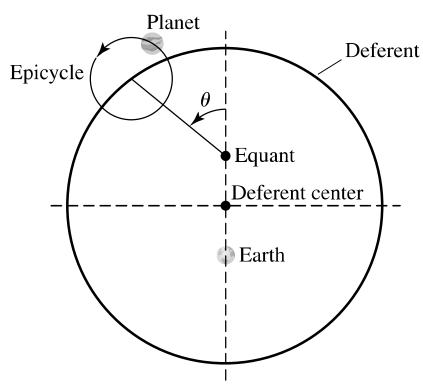
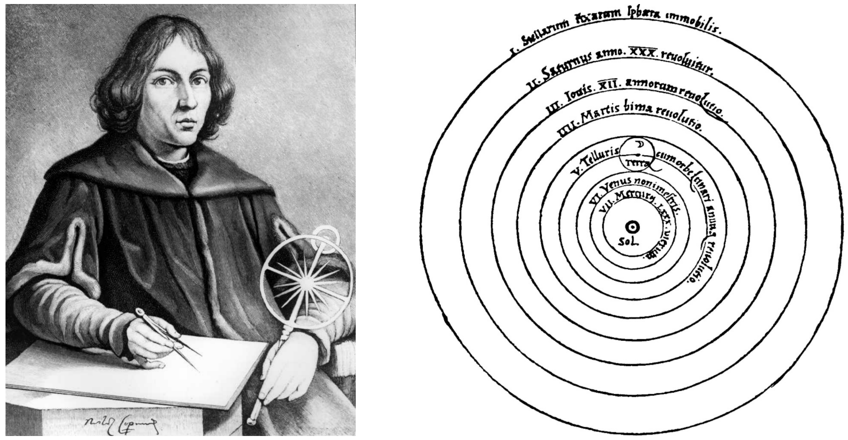


FIGURE 3 The Ptolemaic model of planetary motion.

system by adding **equants** (Fig. 3), resulting in a constant *angular* speed of the epicycle about the deferent ($d\theta/dt$ was assumed to be constant). He also moved Earth away from the deferent center and even allowed for a wobble of the deferent itself. Predictions of the Ptolemaic model did agree more closely with observations than any previously devised scheme, but the original philosophical tenets of Plato (uniform and circular motion) were significantly compromised.

Despite its shortcomings, the Ptolemaic model became almost universally accepted as the correct explanation of the motion of the wandering stars. When a disagreement between the model and observations would develop, the model was modified slightly by the addition of another circle. This process of "fixing" the existing theory led to an increasingly complex theoretical description of observable phenomena.



(a) (b)

FIGURE 4 (a) Nicolaus Copernicus (1473–1543). (b) The Copernican model of planetary motion: Planets travel in circles with the Sun at the center of motion. (Courtesy of Yerkes Observatory.)

2 ■ THE COPERNICAN REVOLUTION

By the sixteenth century the inherent simplicity of the Ptolemaic model was gone. Polish-born astronomer Nicolaus Copernicus (1473–1543), hoping to return the science to a less cumbersome, more elegant view of the universe, suggested a **heliocentric** (Sun-centered) model of planetary motion (Fig. 4).[1] His bold proposal led immediately to a much less complicated description of the relationships between the planets and the stars. Fearing severe criticism from the Catholic Church, whose doctrine then declared that Earth was the center of the universe, Copernicus postponed publication of his ideas until late in life. *De Revolutionibus Orbium Coelestium* (*On the Revolution of the Celestial Sphere*) first appeared in the year of his death. Faced with a radical new view of the universe, along with Earth's location in it, even some supporters of Copernicus argued that the heliocentric model merely represented a mathematical improvement in calculating planetary positions but did not actually reflect the true geometry of the universe. In fact, a preface to that effect was added by Osiander, the priest who acted as the book's publisher.

Bringing Order to the Planets

One immediate consequence of the Copernican model was the ability to establish the order of all of the planets from the Sun, along with their relative distances and orbital periods. The fact that Mercury and Venus are never seen more than 28° and 47°, respectively, east or west of the Sun clearly establishes that their orbits are located inside the orbit of Earth. These planets are referred to as **inferior planets**, and their maximum angular separations east or west of the Sun are known as **greatest eastern elongation** and **greatest western**

[1]Actually, Aristarchus proposed a heliocentric model of the universe in 280 B.C. At that time, however, there was no compelling evidence to suggest that Earth itself was in motion.

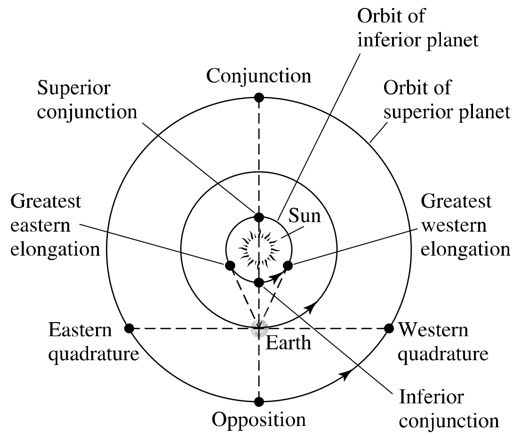


FIGURE 5 Orbital configurations of the planets.

elongation, respectively (see Fig. 5). Mars, Jupiter, and Saturn (the most distant planets known to Copernicus) can be seen as much as 180° from the Sun, an alignment known as **opposition**. This could only occur if these **superior planets** have orbits outside Earth's orbit. The Copernican model also predicts that only inferior planets can pass in front of the solar disk (**inferior conjunction**), as observed.

Retrograde Motion Revisited

The great long-standing problem of astronomy—retrograde motion—was also easily explained through the Copernican model. Consider the case of a superior planet such as Mars. Assuming, as Copernicus did, that the farther a planet is from the Sun, the more slowly it moves in its orbit, Mars will then be overtaken by the faster-moving Earth. As a result, the apparent position of Mars will shift against the relatively fixed background stars, with the planet seemingly moving backward near opposition, where it is closest to Earth and at its brightest (see Fig. 6). Since the orbits of all of the planets are not in the same plane, retrograde loops will occur. The same analysis works equally well for all other planets, superior and inferior.

The relative orbital motions of Earth and the other planets mean that the time interval between successive oppositions or conjunctions can differ significantly from the amount of time necessary to make one complete orbit relative to the background stars (Fig. 7). The former time interval (between oppositions) is known as the **synodic period** (S), and the latter time interval (measured relative to the background stars) is referred to as the **sidereal period** (P). It is left as an exercise to show that the relationship between the two periods is given by

$$1/S = \begin{cases} 1/P - 1/P_\oplus & \text{(inferior)} \\ 1/P_\oplus - 1/P & \text{(superior),} \end{cases} \tag{1}$$

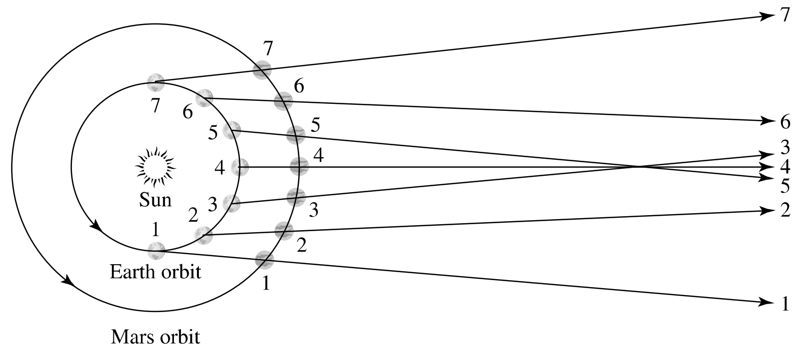


FIGURE 6 The retrograde motion of Mars as described by the Copernican model. Note that the lines of sight from Earth to Mars cross for positions 3, 4, and 5. This effect, combined with the slightly differing planes of the two orbits result in retrograde paths near opposition. Recall the retrograde (or westward) motion of Mars between October 1, 2005, and December 10, 2005, as illustrated in Fig. 2.

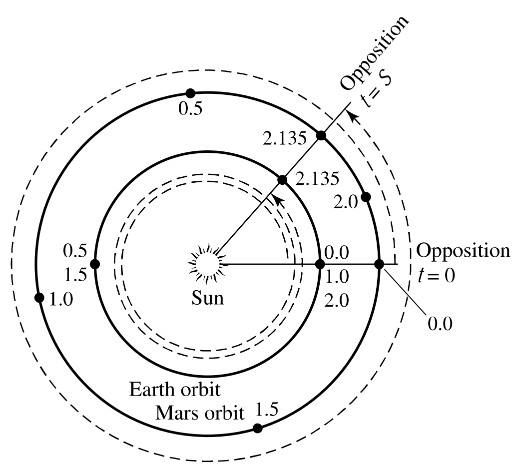


FIGURE 7 The relationship between the sidereal and synodic periods of Mars. The two periods do not agree due to the motion of Earth. The numbers represent the elapsed time in sidereal years since Mars was initially at opposition. Note that Earth completes more than two orbits in a synodic period of $S = 2.135$ yr, whereas Mars completes slightly more than one orbit during one synodic period from opposition to opposition.

when perfectly circular orbits and constant speeds are assumed; $P_{\oplus}$ is the sidereal period of Earth's orbit (365.256308 d).

Although the Copernican model did represent a simpler, more elegant model of planetary motion, it was not successful in predicting positions any more accurately than the Ptolemaic model. This lack of improvement was due to Copernicus's inability to relinquish the 2000-year-old concept that planetary motion required circles, the human notion of perfection. As a consequence, Copernicus was forced (as were the Greeks) to introduce the concept of epicycles to "fix" his model.

Perhaps the quintessential example of a scientific revolution was the revolution begun by Copernicus. What we think of today as the obvious solution to the problem of planetary motion—a heliocentric universe—was perceived as a very strange and even rebellious notion during a time of major upheaval, when Columbus had recently sailed to the "new world" and Martin Luther had proposed radical revisions in Christianity. Thomas Kuhn has suggested that an established scientific theory is much more than just a framework for guiding the study of natural phenomena. The present **paradigm** (or prevailing scientific theory) is actually a way of *seeing* the universe around us. We ask questions, pose new research problems, and interpret the results of experiments and observations in the context of the paradigm. Viewing the universe in any other way requires a complete shift from the current paradigm. To suggest that Earth actually orbits the Sun instead of believing that the Sun inexorably rises and sets about a fixed Earth is to argue for a change in the very structure of the universe, a structure that was believed to be correct and beyond question for nearly 2000 years. Not until the complexity of the old Ptolemaic scheme became too unwieldy could the intellectual environment reach a point where the concept of a heliocentric universe was even possible.

3 ■ POSITIONS ON THE CELESTIAL SPHERE

The Copernican revolution has shown us that the notion of a geocentric universe is incorrect. Nevertheless, with the exception of a small number of planetary probes, our observations of the heavens are still based on a reference frame centered on Earth. The daily (or **diurnal**) rotation of Earth, coupled with its annual motion around the Sun and the slow wobble of its rotation axis, together with relative motions of the stars, planets, and other objects, results in the constantly changing positions of celestial objects. To catalog the locations of objects such as the Crab supernova remnant in Taurus or the great spiral galaxy of Andromeda, coordinates must be specified. Moreover, the coordinate system should not be sensitive to the short-term manifestations of Earth's motions; otherwise the specified coordinates would constantly change.

The Altitude–Azimuth Coordinate System

Viewing objects in the night sky requires only directions to them, not their distances. We can imagine that all objects are located on a celestial sphere, just as the ancient Greeks believed. It then becomes sufficient to specify only two coordinates. The most straightforward coordinate system one might devise is based on the observer's local horizon. The **altitude–azimuth** (or **horizon**) **coordinate system** is based on the measurement of the azimuth angle along the horizon together with the altitude angle above the horizon (Fig. 8). The **altitude** h is defined as that angle measured from the horizon to the object along a great circle[2] that passes through that object and the point on the celestial sphere directly above the observer, a point known as the **zenith**. Equivalently, the **zenith distance** z is the angle measured from the zenith to the object, so $z + h = 90°$. The **azimuth** A is simply the angle

[2]A great circle is the curve resulting from the intersection of a sphere with a plane passing through the *center* of that sphere.

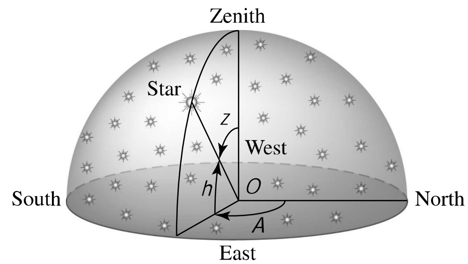


FIGURE 8 The altitude–azimuth coordinate system. h, z, and A are the altitude, zenith distance, and azimuth, respectively.

measured along the horizon eastward from north to the great circle used for the measure of altitude. (The **meridian** is another frequently used great circle; it is defined as passing through the observer's zenith and intersecting the horizon due north and south.)

Although simple to define, the altitude–azimuth system is difficult to use in practice. Coordinates of celestial objects in this system are specific to the local latitude and longitude of the observer and are difficult to transform to other locations on Earth. Also, since Earth is rotating, stars appear to move constantly across the sky, meaning that the coordinates of each object are constantly changing, even for the local observer. Complicating the problem still further, the stars rise approximately 4 minutes earlier on each successive night, so that even when viewed from the same location at a specified time, the coordinates change from day to day.

Daily and Seasonal Changes in the Sky

To understand the problem of these day-to-day changes in altitude–azimuth coordinates, we must consider the orbital motion of Earth about the Sun (see Fig. 9). As Earth orbits the Sun, our view of the distant stars is constantly changing. Our line of sight to the Sun sweeps through the constellations during the seasons; consequently, we see the Sun apparently move through those constellations along a path referred to as the **ecliptic**.[3] During the spring the Sun appears to travel across the constellation of Virgo, in the summer it moves through Orion, during the autumn months it enters Aquarius, and in the winter the Sun is located near Scorpius. As a consequence, those constellations become obscured in the glare of daylight, and other constellations appear in our night sky. This seasonal change in the constellations is directly related to the fact that a given star rises approximately 4 minutes earlier each day. Since Earth completes one sidereal period in approximately 365.26 days, it moves slightly less than 1° around its orbit in 24 hours. Thus Earth must actually rotate nearly 361° to bring the Sun to the meridian on two successive days (Fig. 10). Because of the much greater distances to the stars, they do not shift their positions significantly as Earth orbits the Sun. As a result, placing a star on the meridian on successive nights requires only a 360° rotation. It takes approximately 4 minutes for Earth to rotate the extra 1°. Therefore a given star rises 4 minutes earlier each night. **Solar time** is defined as an *average* interval of

[3]The term *ecliptic* is derived from the observation of eclipses along that path through the heavens.

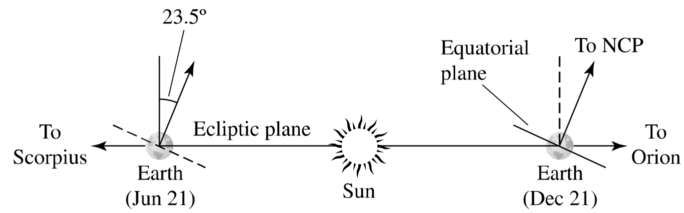


FIGURE 9 The plane of Earth's orbit seen edge-on. The tilt of Earth's rotation axis relative to the ecliptic is also shown.

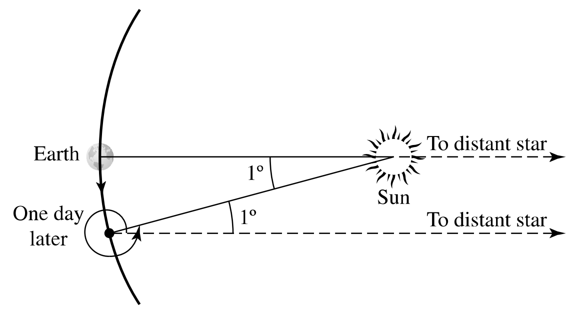


FIGURE 10 Earth must rotate nearly 361° per solar day and only 360° per sidereal day.

24 hours between meridian crossings of the Sun, and **sidereal time** is based on consecutive meridian crossings of a star.

Seasonal climatic variations are also due to the orbital motion of Earth, coupled with the approximately 23.5° tilt of its rotation axis. As a result of the tilt, the ecliptic moves north and south of the **celestial equator** (Fig. 11), which is defined by passing a plane through Earth at its equator and extending that plane out to the celestial sphere. The sinusoidal shape of the ecliptic occurs because the Northern Hemisphere alternately points toward and then away from the Sun during Earth's annual orbit. Twice during the year the Sun crosses the celestial equator, once moving northward along the ecliptic and later moving to the south. In the first case, the point of intersection is called the **vernal equinox** and the southern crossing occurs at the **autumnal equinox**. Spring officially begins when the center of the Sun is precisely on the vernal equinox; similarly, fall begins when the center of the Sun crosses the autumnal equinox. The most northern excursion of the Sun along the ecliptic occurs at the **summer solstice**, representing the official start of summer, and the southernmost position of the Sun is defined as the **winter solstice**.

The seasonal variations in weather are due to the position of the Sun relative to the celestial equator. During the summer months in the Northern Hemisphere, the Sun's northern declination causes it to appear higher in the sky, producing longer days and more intense sunlight. During the winter months the declination of the Sun is below the celestial equator, its path above the horizon is shorter, and its rays are less intense (see Fig. 12). The more direct the Sun's rays, the more energy per unit area strikes Earth's surface and the higher the resulting surface temperature.

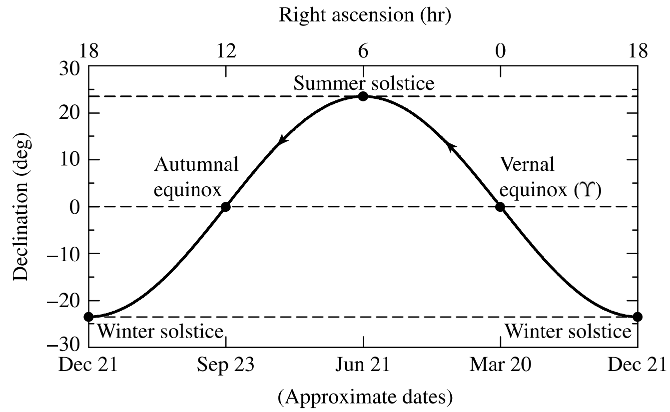


FIGURE 11 The ecliptic is the annual path of the Sun across the celestial sphere and is sinusoidal about the celestial equator. Summer solstice is at a declination of 23.5° and winter solstice is at a declination of −23.5°. See Fig. 13 for explanations of right ascension and declination.

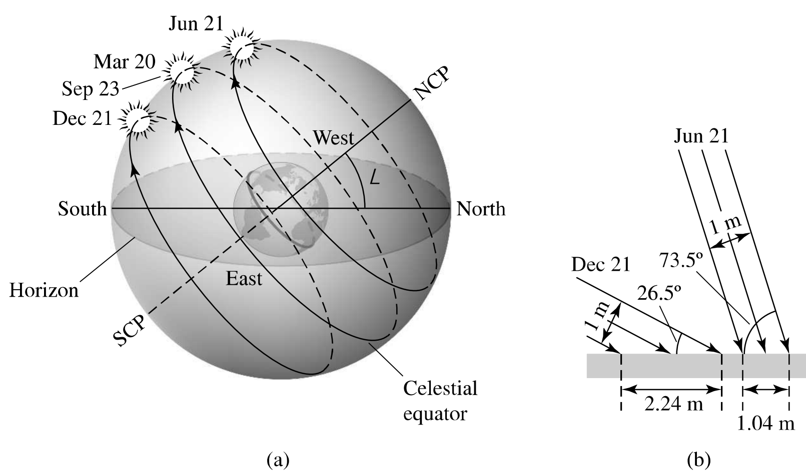


FIGURE 12 (a) The diurnal path of the Sun across the celestial sphere for an observer at latitude L when the Sun is located at the vernal equinox (March), the summer solstice (June), the autumnal equinox (September), and the winter solstice (December). NCP and SCP designate the north and south celestial poles, respectively. The dots represent the location of the Sun at local noon on the approximate dates indicated. (b) The direction of the Sun's rays at noon at the summer solstice (approximately June 21) and at the winter solstice (approximately December 21) for an observer at 40° N latitude.

The Equatorial Coordinate System

A coordinate system that results in nearly constant values for the positions of celestial objects, despite the complexities of diurnal and annual motions, is necessarily less straightforward than the altitude–azimuth system. The **equatorial coordinate system** (see Fig. 13) is based on the latitude–longitude system of Earth but does not participate in the planet's rotation. **Declination** δ is the equivalent of latitude and is measured in degrees north or

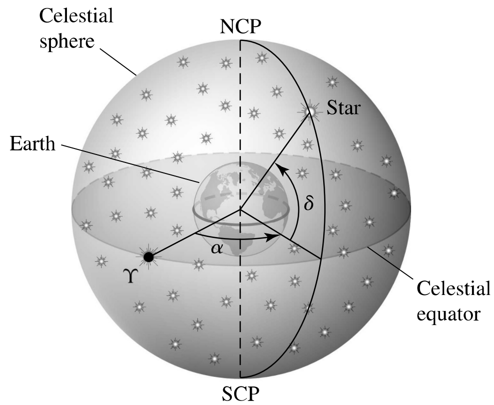


FIGURE 13 The equatorial coordinate system. α, δ, and ϒ designate right ascension, declination, and the position of the vernal equinox, respectively.

south of the celestial equator. **Right ascension** α is analogous to longitude and is measured eastward along the celestial equator from the vernal equinox (ϒ) to its intersection with the object's **hour circle** (the great circle passing through the object being considered and through the north celestial pole). Right ascension is traditionally measured in hours, minutes, and seconds; 24 hours of right ascension is equivalent to 360°, or 1 hour = 15°. The rationale for this unit of measure is based on the 24 hours (sidereal time) necessary for an object to make two successive crossings of the observer's local meridian. The coordinates of right ascension and declination are also indicated in Figs. 2 and 11. Since the equatorial coordinate system is based on the celestial equator and the vernal equinox, changes in the latitude and longitude of the observer do not affect the values of right ascension and declination. Values of α and δ are similarly unaffected by the annual motion of Earth around the Sun.

The **local sidereal time** of the observer is defined as the amount of time that has elapsed since the vernal equinox last traversed the meridian. Local sidereal time is also equivalent to the **hour angle** H of the vernal equinox, where hour angle is defined as the angle between a celestial object and the observer's meridian, measured in the direction of the object's motion around the celestial sphere.

Precession

Despite referencing the equatorial coordinate system to the celestial equator and its intersection with the ecliptic (the vernal equinox), **precession** causes the right ascension and declination of celestial objects to change, albeit very slowly. Precession is the slow wobble of Earth's rotation axis due to our planet's nonspherical shape and its gravitational interaction with the Sun and the Moon. It was Hipparchus who first observed the effects of precession. Although we will not discuss the physical cause of this phenomenon in detail, it is completely analogous to the well-known precession of a child's toy top. Earth's precession period is 25,770 years and causes the north celestial pole to make a slow circle through the heavens. Although Polaris (the North Star) is currently within 1° of the north celestial pole, in 13,000 years it will be nearly 47° away from that point. The same effect also causes a $50.26'' \text{ yr}^{-1}$ westward motion of the vernal equinox along the ecliptic.[4] An additional precession effect due to Earth–planet interactions results in an eastward motion of the vernal equinox of $0.12'' \text{ yr}^{-1}$.

Because precession alters the position of the vernal equinox along the ecliptic, it is necessary to refer to a specific **epoch** (or reference date) when listing the right ascension and declination of a celestial object. The current values of α and δ may then be calculated, based on the amount of time elapsed since the reference epoch. The epoch commonly used today for astronomical catalogs of stars, galaxies, and other celestial phenomena refers to an object's position at noon in Greenwich, England (**universal time**, **UT**) on January 1, 2000.[5] A catalog using this reference date is designated as J2000.0. The prefix, J, in the designation J2000.0 refers to the **Julian calendar**, which was introduced by Julius Caesar in 46 B.C.

Approximate expressions for the changes in the coordinates relative to J2000.0 are

$$\Delta\alpha = M + N \sin\alpha \tan\delta \tag{2}$$

$$\Delta\delta = N \cos\alpha, \tag{3}$$

where M and N are given by

$$M = 1^\circ\!.2812323T + 0^\circ\!.0003879T^2 + 0^\circ\!.0000101T^3$$

$$N = 0^\circ\!.5567530T - 0^\circ\!.0001185T^2 - 0^\circ\!.0000116T^3$$

and T is defined as

$$T = (t - 2000.0)/100 \tag{4}$$

where t is the current date, specified in fractions of a year.

Example 3.1. Altair, the brightest star in the summer constellation of Aquila, has the following J2000.0 coordinates: $\alpha = 19^{\text{h}}50^{\text{m}}47.0^{\text{s}}$, $\delta = +08^\circ 52' 06.0''$. Using Eqs. (2) and (3), we may precess the star's coordinates to noon Greenwich mean time on July 30, 2005. Writing the date as $t = 2005.575$, we have that $T = 0.05575$. This implies that $M = 0.071430^\circ$ and $N = 0.031039^\circ$. From the relations between time and the angular

continued

[4] 1 arcminute = 1′ = 1/60 degree; 1 arcsecond = 1″ = 1/60 arcminute.

[5] Universal time is also sometimes referred to as **Greenwich mean time**. Technically there are two forms of universal time; **UT1** is based on Earth's rotation rate, and **UTC** (**coordinated universal time**) is the basis of the worldwide system of civil time and is measured by atomic clocks. Because Earth's rotation rate is less regular than the time kept by atomic clocks, it is necessary to adjust UTC clocks by about one second (a *leap second*) roughly every year to year and a half. Among other effects contributing to the difference between UT1 and UTC is the slowing of Earth's rotation rate due to tidal effects.

measure of right ascension,

$$1^{\mathrm{h}} = 15^\circ$$
$$1^{\mathrm{m}} = 15'$$
$$1^{\mathrm{s}} = 15''$$

the corrections to the coordinates are

$$\begin{aligned}\Delta\alpha &= 0.071430^\circ + (0.031039^\circ) \sin 297.696^\circ \tan 8.86833^\circ \\ &= 0.067142^\circ \simeq 16.11^{\mathrm{s}}\end{aligned}$$

and

$$\begin{aligned}\Delta\delta &= (0.031039^\circ) \cos 297.696^\circ \\ &= 0.014426^\circ \simeq 51.93''.\end{aligned}$$

Thus Altair's precessed coordinates are $\alpha = 19^{\mathrm{h}}51^{\mathrm{m}}03.1^{\mathrm{s}}$ and $\delta = +08^\circ 52' 57.9''$.

Measurements of Time

The civic calendar commonly used in most countries today is the **Gregorian calendar**. The Gregorian calendar, introduced by Pope Gregory XIII in 1582, carefully specifies which years are to be considered leap years. Although leap years are useful for many purposes, astronomers are generally interested in the number of days (or seconds) between events, not in worrying about the complexities of leap years. Consequently, astronomers typically refer to the times when observations were made in terms of the elapsed time since some specified zero time. The time that is universally used is noon on January 1, 4713 B.C., as specified by the Julian calendar. This time is designated as JD 0.0, where JD indicates **Julian Date**.[6] The Julian date of J2000.0 is JD 2451545.0. Times other than noon universal time are specified as fractions of a day; for example, 6 PM January 1, 2000 UT would be designated JD 2451545.25. Referring to Julian date, the parameter T defined by Eq. (4) can also be written as

$$T = (\mathrm{JD} - 2451545.0)/36525,$$

where the constant 36,525 is taken from the **Julian year**, which is defined to be exactly 365.25 days.

Another commonly-used designation is the **Modified Julian Date** (**MJD**), defined as MJD $\equiv$ JD $-$ 2400000.5, where JD refers to the Julian date. Thus a MJD day begins at midnight, universal time, rather than at noon.

[6]The Julian date JD 0.0 was proposed by Joseph Justus Scaliger (1540–1609) in 1583. His choice was based on the convergence of three calendar cycles; the 28 years required for the Julian calendar dates to fall on the same days of the week, the 19 years required for the phases of the Moon to *nearly* fall on the same dates of the year, and the 15-year Roman tax cycle. $28 \times 19 \times 15 = 7980$ means that the three calendars align once every 7980 years. JD 0.0 corresponds to the last time the three calendars all started their cycles together.

Because of the need to measure events very precisely in astronomy, various high-precision time measurements are used. For instance, **Heliocentric Julian Date** (HJD) is the Julian Date of an event as measured from the center of the Sun. In order to determine the heliocentric Julian date, astronomers must consider the time it would take light to travel from a celestial object to the center of the Sun rather than to Earth. **Terrestrial Time** (TT) is time measured on the surface of Earth, taking into consideration the effects of special and general relativity as Earth moves around the Sun and rotates on its own axis.

Archaeoastronomy

An interesting application of the ideas discussed above is in the interdisciplinary field of **archaeoastronomy**, a merger of archaeology and astronomy. Archaeoastronomy is a field of study that relies heavily on historical adjustments that must be made to the positions of objects in the sky resulting from precession. It is the goal of archaeoastronomy to study the astronomy of past cultures, the investigation of which relies heavily on the alignments of ancient structures with celestial objects. Because of the long periods of time since construction, care must be given to the proper precession of celestial coordinates if any proposed alignments are to be meaningful. The Great Pyramid at Giza (Fig. 14), one of the "seven wonders of the world," is an example of such a structure. Believed to have been erected about 2600 B.C., the Great Pyramid has long been the subject of speculation. Although many of the proposals concerning this amazing monument are more than somewhat fanciful, there can be no doubt about its careful orientation with the four cardinal positions, north, south, east, and west. The greatest misalignment of any side from a true cardinal direction is no more than $5\frac{1}{2}'$. Equally astounding is the nearly perfect square formed by its base; no two sides differ in length by more than 20 cm.

Perhaps the most demanding alignments discovered so far are associated with the "air shafts" leading from the King's Chamber (the main chamber of the pyramid) to the outside. These air shafts seem too poorly designed to circulate fresh air into the tomb of Pharaoh, and

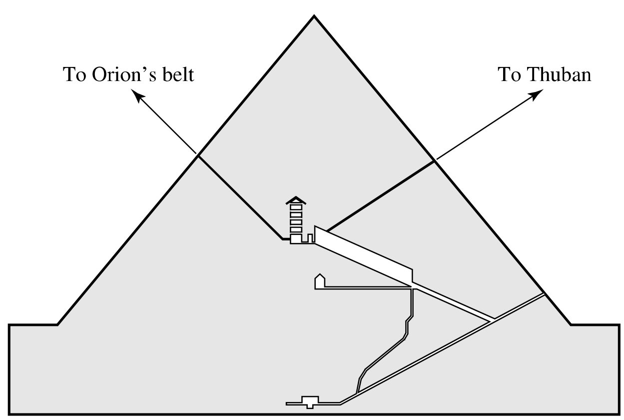


FIGURE 14 The astronomical alignments of the Great Pyramid at Giza. (Adaptation of a figure from Griffith Observatory.)

it is now thought that they served another function. The Egyptians believed that when their pharaohs died, their souls would travel to the sky to join Osiris, the god of life, death, and rebirth. Osiris was associated with the constellation we now know as Orion. Allowing for over one-sixth of a precession period since the construction of the Great Pyramid, Virginia Trimble has shown that one of the air shafts pointed directly to Orion's belt. The other air shaft pointed toward Thuban, the star that was *then* closest to the north celestial pole, the point in the sky about which all else turns.

As a modern scientific culture, we trace our study of astronomy to the ancient Greeks, but it has become apparent that many cultures carefully studied the sky and its mysterious points of light. Archaeological structures worldwide apparently exhibit astronomical alignments. Although some of these alignments may be coincidental, it is clear that many of them were by design.

The Effects of Motions Through the Heavens

Another effect contributing to the change in equatorial coordinates is due to the intrinsic velocities of the objects themselves.[7] As we have already discussed, the Sun, the Moon, and the planets exhibit relatively rapid and complex motions through the heavens. The stars also move with respect to one another. Even though their actual speeds may be very large, the apparent relative motions of stars are generally very difficult to measure because of their enormous distances.

Consider the velocity of a star relative to an observer (Fig. 15). The velocity vector may be decomposed into two mutually perpendicular components, one lying along the line of sight and the other perpendicular to it. The line-of-sight component is the star's **radial velocity**, $\mathbf{v}_r$; the second component is the star's

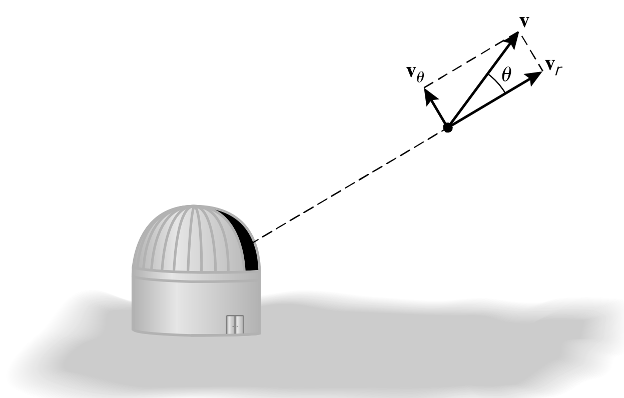


FIGURE 15 The components of velocity. $\mathbf{v}_r$ is the star's radial velocity and $\mathbf{v}_\theta$ is the star's transverse velocity.

[7]Parallax is an important periodic motion of the stars resulting from the motion of Earth about the Sun.

transverse or **tangential velocity**, $\mathbf{v}_\theta$, along the celestial sphere. This transverse velocity appears as a *slow, angular change* in its equatorial coordinates, known as **proper motion** (usually expressed in seconds of arc per year). In a time interval Δt, the star will have moved in a direction perpendicular to the observer's line of sight a distance

$$\Delta d = v_\theta \Delta t.$$

If the distance from the observer to the star is r, then the angular change in its position along the celestial sphere is given by

$$\Delta\theta = \frac{\Delta d}{r} = \frac{v_\theta}{r}\Delta t.$$

Thus the star's proper motion, μ, is related to its transverse velocity by

$$\boxed{\mu \equiv \frac{d\theta}{dt} = \frac{v_\theta}{r}.} \tag{5}$$

An Application of Spherical Trigonometry

The laws of spherical trigonometry must be employed in order to find the relationship between $\Delta\theta$ and changes in the equatorial coordinates, $\Delta\alpha$ and $\Delta\delta$, on the celestial sphere. A spherical triangle such as the one depicted in Fig. 16 is composed of three intersecting segments of great circles. For a spherical triangle the following relationships hold (with all sides measured in arc length, e.g., degrees):

Law of sines

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

Law of cosines for sides

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

Law of cosines for angles

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

Figure 17 shows the motion of a star on the celestial sphere from point A to point B. The angular distance traveled is $\Delta\theta$. Let point P be located at the north celestial pole so that the arcs AP, AB, and BP form segments of great circles. The star is then said to be moving in the direction of the **position angle** ϕ ($\angle PAB$), measured from the north celestial pole. Now, construct a segment of a circle NB such that N is at the same declination as B and $\angle PNB = 90^\circ$. If the coordinates of the star at point A are (α, δ) and its new coordinates at point B are $(\alpha + \Delta\alpha, \delta + \Delta\delta)$, then $\angle APB = \Delta\alpha$, $\overline{AP} = 90^\circ - \delta$, and $\overline{NP} = \overline{BP} = 90^\circ - (\delta + \Delta\delta)$. Using the law of sines,

$$\frac{\sin(\Delta\theta)}{\sin(\Delta\alpha)} = \frac{\sin[90^\circ - (\delta + \Delta\delta)]}{\sin\phi},$$

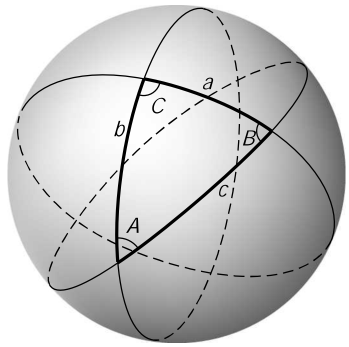


FIGURE 16 A spherical triangle. Each leg is a segment of a great circle on the surface of a sphere, and all angles are less than 180°. a, b, and c are in angular units (e.g., degrees).

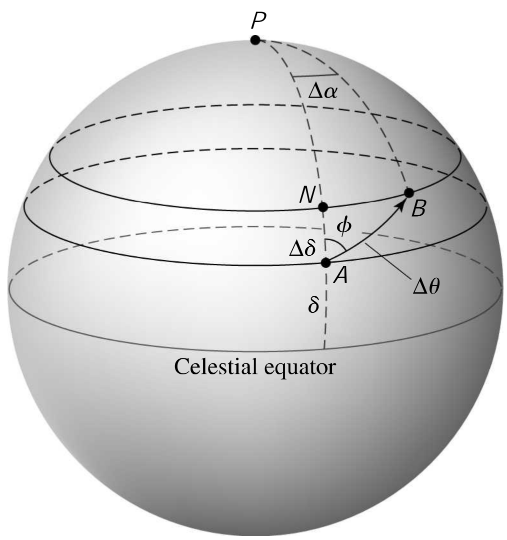


FIGURE 17 The proper motion of a star across the celestial sphere. The star is assumed to be moving from A to B along the position angle ϕ.

or

$$\sin(\Delta\alpha)\cos(\delta + \Delta\delta) = \sin(\Delta\theta)\sin\phi.$$

Assuming that the changes in position are much less than one radian, we may use the small-angle approximations $\sin\epsilon \sim \epsilon$ and $\cos\epsilon \sim 1$. Employing the appropriate trigonometric identity and neglecting all terms of second order or higher, the previous equation reduces to

$$\Delta\alpha = \Delta\theta \frac{\sin\phi}{\cos\delta}. \tag{6}$$

The law of cosines for sides may also be used to find an expression for the change in the declination:

$$\cos\left[90^\circ - (\delta + \Delta\delta)\right] = \cos\left(90^\circ - \delta\right)\cos\left(\Delta\theta\right) + \sin\left(90^\circ - \delta\right)\sin\left(\Delta\theta\right)\cos\phi.$$

Again using small-angle approximations and trigonometric identities, this expression reduces to

$$\Delta\delta = \Delta\theta\cos\phi. \tag{7}$$

(Note that this is the same result that would be obtained if we had used plane trigonometry. This should be expected, however, since we have assumed that the triangle being considered has an area much smaller than the total area of the sphere and should therefore appear essentially flat.) Combining Eqs. (6) and (7), we arrive at the expression for the angular distance traveled in terms of the changes in right ascension and declination:

$$\boxed{(\Delta\theta)^2 = (\Delta\alpha\cos\delta)^2 + (\Delta\delta)^2.} \tag{8}$$

4 ■ PHYSICS AND ASTRONOMY

The mathematical view of nature first proposed by Pythagoras and the Greeks led ultimately to the Copernican revolution. The inability of astronomers to accurately fit the observed positions of the "wandering stars" with mathematical models resulted in a dramatic change in our perception of Earth's location in the universe. However, an equally important step still remained in the development of science: the search for *physical causes* of observable phenomena. As we will see, the modern study of astronomy relies heavily on an understanding of the physical nature of the universe. The application of physics to astronomy, *astrophysics*, has proved very successful in explaining a wide range of observations, including strange and exotic objects and events, such as pulsating stars, supernovae, variable X-ray sources, black holes, quasars, gamma-ray bursts, and the Big Bang.

As a part of our investigation of the science of astronomy, it will be necessary to study the details of celestial motions, the nature of light, the structure of the atom, and the shape of space itself. Rapid advances in astronomy over the past several decades have occurred because of advances in our understanding of fundamental physics and because of improvements in the tools we use to study the heavens: telescopes and computers.

Essentially every area of physics plays an important role in some aspect of astronomy. Particle physics and astrophysics merge in the study of the Big Bang; the basic question of the origin of the zoo of elementary particles, as well as the very nature of the fundamental forces, is intimately linked to how the universe was formed. Nuclear physics provides information about the types of reactions that are possible in the interiors of stars, and atomic physics describes how individual atoms interact with one another and with light, processes that are basic to a great many astrophysical phenomena. Condensed-matter physics plays a