1.1 ***Set:*** Any collection of well-defined objects.

1.2 ***Elements:*** Objects belonging to a set.

1.3 ***Empty set:*** Set having no elements and is denoted Ø.

1.4 ***Equal sets:*** Two sets A and B are said to be equal, if they contain same elements or every element of A belong to B and vice-versa.

1.5 ***Finite set:*** A set having definite number of elements is called *finite* set. A set which is not a finite set is called *infinite* set.

1.6 ***Family or class of sets:*** A set whose numbers are family of sets or class of sets. Family of sets or class of sets are denoted by script letters $\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, P$ etc.

1.7 ***Indexed family of sets:*** A family C of sets is called indexed family if there exists a set I such that for each element $i \in I$, there exists unique member $A \in C$ associated with i. In this case the set I is called index set, C is called indexed family sets and we write $C = \{A_i : i \in I\}$.

1.8 ***Intervals:*** Let a, b be real numbers and $a < b$. Then

$$(a, b) = \{x \in \mathbb{R} | a < x < b\}$$

$$[a, b) = \{x \in \mathbb{R} | a \le x < b\}$$

$$(a, b] = \{x \in \mathbb{R} | a < x \le b\}$$

$$[a, b] = \{x \in \mathbb{R} | a \le x \le b\}$$

$$(-\infty, +\infty) \quad \text{or} \quad (-\infty, \infty) \text{ is } \mathbb{R}$$

1.9 ***Subset and superset:*** A set A is called a subset of a set B, if every element of A is also an element of B. In this case we write $A \subseteq B$. If A is a subset of B, then B is called superset of A. If A is not a subset of B, then we write $A \not\subseteq B$.

1.10 ***Proper subset:*** Set A is called a proper subset of a set B if A is a subset of B and is not equal to B.

1.11 ***Powerset:*** If X is a set, then the collection of all subsets of X is called the powerset of X and is denoted by $P(X)$.

1.12 ***Cardinality of a set:*** If X is a finite set having n elements, then n called cardinality of X and is denoted by $|X|$ or $n(X)$.

1.13 If $|X| = n$, then $|P(X)| = 2^n$.

1.14 ***Intersection of sets:*** For any two sets A and B, the intersection of A and B is the set of all elements belonging to both A and B and is denoted by

$$A \cap B = \{x | x \in A \text{ and } x \in B\}$$

1.15 ***Theorem:*** The following hold for any sets A, B and C.

(1) $A \subseteq B \Leftrightarrow A = A \cap B$

(2) $A \cap A = A$

(3) $A \cap B = B \cap A$ (Commutative law)

(4) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(5) $A \cap \varnothing = \varnothing$, where Ø is the empty set.

(6) For any set X, $X \subseteq A \cap B \Leftrightarrow X \subseteq A$ and $X \subseteq B$.

(7) In view of (4) we write $A \cap B \cap C$ for $A \cap (B \cap C)$.

(8) For any sets $A_1, A_2, \ldots, A_n$ we write $\bigcap_{i=1}^{n} A_i$ for $A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$.

1.16 ***Disjoint sets:*** Two sets A and B are said to be disjoint sets if $A \cap B = \varnothing$.

1.17 ***Union of sets:*** For any two sets A and B, their union is defined to be the set of all elements belonging to either A or to B and this set is denoted by $A \cup B$. That is $A \cup B = \{x | x \in A \text{ or } x \in B\}$.

1.18 ***Theorem:*** For any sets A, B and C the following hold.

(1) $A \cap B \subseteq A \cup B$

(2) For any set X, $A \cup B \subseteq X \Leftrightarrow A \subseteq X$ and $B \subseteq X$

(3) $A \cup A = A$

(4) $A \cup B = B \cup A$ (Commutative law)

(5) $(A \cup B) \cup C = A \cup (B \cup C)$ and we write $A \cup B \cup C$ for $(A \cup B) \cup C$

(6) $A \cap B = A \Leftrightarrow A \cup B = B$

(7) $A \cup \varnothing = A$

(8) $A \cap (A \cup B) = A$

(9) $A \cup (A \cap B) = A$

1.19 ***Theorem (Distributive laws):*** If A, B and C are three sets, then

(1) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(2) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

1.20 ***Theorem:*** For any sets A, B and C, $A \cap B = A \cap C$ and $A \cup B = A \cup C \Rightarrow B = C$.

1.21 If $\{A_i\}_{i\in I}$ is an indexed family of sets then $\bigcup_{i\in I} A_i$ is the set of all elements x where x belongs to atleast one A_i.

1.22 ***Set difference:*** For any two sets A and B, $A - B = \{x \in A \mid x \notin B\} = A - (A \cap B)$

1.23 ***De Morgan's laws:*** If A, B and C are any sets, then

(1) $A - (B \cup C) = (A - B) \cap (A - C)$

(2) $A - (B \cap C) = (A - B) \cup (A - C)$

1.24 ***Theorem:*** Let A, B and C be sets. Then

(1) $B \subseteq C \Rightarrow A - C \subseteq A - B$

(2) $A \subseteq B \Rightarrow A - C \subseteq B - C$

(3) $(A \cup B) - C = (A - C) \cup (B - C)$

(4) $(A \cap B) - C = (A - C) \cap (B - C)$

(5) $(A - B) - C = A - (B \cup C) = (A - B) \cap (A - C)$

(6) $A - (B - C) = (A - B) \cup (A \cap C)$

1.25 ***Universal set:*** If $\{A_i\}_{i\in I}$ is a class of sets, then the set $X = \bigcup_{i\in I} A_i$ is called universal set. In fact the set X whose subsets are under our consideration is called universal set.

Caution: Do not be mistaken that universal set means the set which contains all objects in the universe. Do not be carried away with word universal. In fact, the fundamental axiom of set theory is:

Given any set, there is always an element which does not belong to the given set.

1.26 ***Complement of a set:*** If X is an universal set and $A \subseteq X$ then the set $X - A$ is called complement of A and is denoted by A' or A^c.

1.27 ***Relative complement:*** If X is an universal set and A, B are subsets of X, then $A - B = A \cap B'$ is called relative complement of B in A.

1.28 ***De Morgan's laws (General form):*** If A and B are two sets, then

(1) $(A \cup B)' = A' \cap B'$

(2) $(A \cap B)' = A' \cup B'$

1.29 ***Symmetric difference:*** For any two sets A and B, the set $(A - B) \cup (B - A)$ is called symmetric difference of A and B and is denoted by $A \Delta B$. Since $A - B = A \cap B'$ and $B - A = B \cap A'$, $A \Delta B = (A \cap B') \cup (B \cap A')$.

1.30 ***Theorem:*** The following hold for any sets A, B and C.

(1) $A \Delta B = B \Delta A$ (Commutative law)

(2) $(A \Delta B) \Delta C = A \Delta (B \Delta C)$ (Associative law)

(3) $A \Delta \emptyset = A$

(4) $A \Delta A = \emptyset$

1.31 ***Theorem:*** If A and B are disjoint sets, then

(1) $n(A \cup B) = n(A) + n(B)$

(2) If $A_1, A_2, \ldots, A_m$ are pairwise disjoint sets, then

$$n\left(\bigcup_{i=1}^{m} A_i\right) = n(A_1) + n(A_2) + \cdots + n(A_m)$$

Recall that for any finite set P, $n(P)$ denotes the number of elements in P.

1.32 ***Theorem:*** For any finite sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

1.33 ***Theorem:*** For any finite sets A, B and C,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

1.34 ***Theorem:*** If A, B and C are finite sets, then the number of elements belonging to exactly two of the sets is

$$n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$$

1.35 ***Theorem:***

(1) If A, B and C are finite sets, then the number of elements belonging to exactly one of the sets is

$$n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + n(A \cap B \cap C)$$

(2) If A and B are finite sets, then the number of elements belonging to exactly one of the sets equals

$$n(A \Delta B) = n(A) + n(B) - 2n(A \cap B) = n(A \cup B) - n(A \cap B)$$

Relations

1.36 ***Ordered pair:*** A pair of elements written in a particular order is called an ordered pair and is written by listing its two elements in a particular order, separated by a comma and enclosing the pair in brackets. In the ordered pair (x, y), x is the first element called first component and y is the second element called second component. Also x is called first coordinate and y is called second coordinate.

1.37 ***Cartesian product:*** If A and B are sets, then the set of all ordered pairs (a, b) with $a \in A$ and $b \in B$ is called the Cartesian product of A and B and is denoted by $A \times B$ (read as A cross B). That is

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

1.38 Let A, B be any sets and $\emptyset$ is the empty set. Then

(1) $A \times B = \emptyset \Leftrightarrow A = \emptyset$ or $B = \emptyset$.

(2) If one of A and B is an infinite set and the other is a non-empty set, then $A \times B$ is an infinite set.

(3) $A \times B = B \times A \Leftrightarrow A = B$.

1.39 ***Cartesian product of n sets (n is a finite positive integer greater than or equal to 2):*** Let $A_1, A_2, A_3, \ldots, A_n$ be n sets. Then their Cartesian product is defined to be the set of all n-tuples $(a_1, a_2, \ldots, a_n)$ such that $a_i \in A_i$ for $i = 1, 2, 3, \ldots, n$ and is denoted by

$$A_1 \times A_2 \times A_3 \times \cdots \times A_n \quad \text{or} \quad \mathop{\mathbf{X}}_{i=1}^{n} A_i \quad \text{or} \quad \prod_{i=1}^{n} A_i$$

That is,

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } 1 \le i \le n\}$$

The Cartesian product of a set A with itself n times is denoted by A^n.

1.40 ***Theorem:*** If A and B are finite sets, then $n(A \times B) = n(A) \cdot n(B)$. In general, if $A_1, A_2, \ldots, A_m$ are infinite sets, then $n(A_1 \times A_2 \times \cdots \times A_m) = n(A_1) \times n(A_2) \times \cdots \times n(A_m)$. In particular, $n(A^m) = (n(A))^m$ where A is a finite set.

1.41 ***Theorem:*** Let A, B, C and D be any sets. Then

(1) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(2) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

(3) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(4) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

(5) $(A \cup B) \times (C \cup D) = (A \times C) \cup (A \times D) \cup (B \times C) \cup (B \times D)$

(6) $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D) = (A \times D) \cap (B \times C)$

(7) $(A - B) \times C = (A \times C) - (B \times C)$

(8) $A \times (B - C) = (A \times B) - (A \times C)$

1.42 ***Relation:*** For any two sets A and B, any subset of $A \times B$ is called a relation from A to B.

1.43 ***Symbol aRb:*** Let R be a relation from a set A to a set B ($\mathrm{R} \subseteq A \times B$). If $(a, b) \in \mathrm{R}$, then a is said to be R related to b or a is said to be related to b and we write $a\mathrm{R}b$.

1.44 ***Domain:*** Let R be a relation from a set A to a set B. Then the set of all first components of the ordered pairs belonging to R is called the domain of R and is denoted by Dom(R).

1.45 ***Range:*** If R is a relation from a set A to a set B, then the set of all second components of the ordered pairs belonging to R is called range of R and is denoted by Range(R).

1.46 ***Theorem:*** If A and B are finite non-empty sets such that $n(A) = m$ and $n(B) = n$, then the number of relations from A to B is 2^{mn} which include the empty set and the whole set $A \times B$.

1.47 ***Relation on a set:*** If A is a set, then any subset of $A \times A$ is called a binary relation on A or simply a relation on A.

1.48 ***Composition of relations:*** Let A, B and C be sets, R is a relation from A to B and S is a relation from B to C. Then, the composition of R and S denoted by $\mathrm{S} \circ \mathrm{R}$ defined to be

$$\mathrm{S} \circ \mathrm{R} = \{(a, c) \in A \times C \,|\, \text{there exist } b \in B \text{ such that } (a, b) \in \mathrm{R} \text{ and } (b, c) \in \mathrm{S}\}$$

1.49 ***Theorem:*** Let A, B and C be sets, R a relation from A to B and S a relation from B to C. Then the following hold:

(1) $\mathrm{S} \circ \mathrm{R} \ne \emptyset$ if and only if $\mathrm{Range(R)} \cap \mathrm{Dom(S)} \ne \emptyset$

(2) $\mathrm{Dom(S \circ R)} = \mathrm{Dom(R)}$

(3) $\mathrm{Range(S \circ R)} \subseteq \mathrm{Range(S)}$

1.50 ***Theorem:*** Let A, B, C and D be non-empty sets, $R \subseteq A \times B$, $S \subseteq B \times C$ and $T \subseteq C \times D$. Then

$$(\mathrm{T} \circ \mathrm{S}) \circ \mathrm{R} = \mathrm{T} \circ (\mathrm{S} \circ \mathrm{R}) \text{ (Associative law)}$$

1.51 ***Inverse relation:*** Let A and B be non-empty sets and R a relation from A to B. Then the inverse of R is defined as the set $\{(b, a) \in B \times A \,|\, (a, b) \in \mathrm{R}\}$ and is denoted by R^{-1}.

1.52 ***Theorem:*** Let A, B and C be non-empty sets, R a relation from A to B and S a relation from B to C. Then the following hold:

(1) $(\mathrm{S} \circ \mathrm{R})^{-1} = \mathrm{R}^{-1} \circ \mathrm{S}^{-1}$

(2) $(\mathrm{R}^{-1})^{-1} = \mathrm{R}$

Types of Relations

1.53 ***Reflexive relation:*** Let X be a non-empty set and R relation from X to X. Then R is said to be reflexive on X if $(x, x) \in \mathrm{R}$ for all $x \in X$.

1.54 ***Symmetric relation:*** A relation R on a non-empty set X is called symmetric if $(x, y) \in \mathrm{R} \Rightarrow (y, x) \in \mathrm{R}$.

1.55 ***Transitive relation:*** A relation R on a non-empty set X is called transitive if $(x, y) \in \text{R}$ and $(y, z) \in \text{R} \Rightarrow (x, z) \in \text{R}$.

1.56 ***Equivalence relation:*** A relation R on a non-empty set X is called an equivalence relation if it is reflexive, symmetric and transitive.

1.57 ***Partition of a set:*** Let X be a non-empty set. A class of subsets of X is called a partition of X if they are pairwise disjoint and their union is X.

1.58 ***Equivalence class:*** Let X be a non-empty set and R an equivalence relation on X. If $x \in X$, then the set $\{y \in X | (x, y) \in \text{R}\}$ is called the equivalence class of x with respect to R or the R-equivalence of x or simply the R-class of x and is denoted by $\text{R}(x)$.

1.59 ***Theorem:*** Let R be an equivalence relation on a set X and $a, b \in X$. Then the following statements are equivalent:

(1) $(a, b) \in \text{R}$

(2) $\text{R}(a) = \text{R}(b)$

(3) $\text{R}(a) \cap \text{R}(b) \neq \emptyset$

1.60 ***Theorem:*** Let R be an equivalence relation on X. Then the class of all R-classes form a partition of X.

1.61 ***Theorem:*** Let X be a non-empty and $\{A_i\}_{i \in I}$ a partition of X. Then

$$\text{R} = \{(x, y) \in X \times X \,|\, \text{both } x \text{ and } y \text{ belong to the same } A_i, i \in I\}$$

is an equivalence relation on X, whose R-classes are precisely A_i's.

1.62 ***Theorem:*** Let R and S be equivalence relations on a non-empty X. Then $\text{R} \cap \text{S}$ is also an equivalence relation on X and for any $x \in X$, $(\text{R} \cap \text{S})(x) = \text{R}(x) \cap \text{S}(x)$.

1.63 ***Theorem:*** Let R and S be equivalence relations on a set X. Then the following statements are equivalent.

(1) $\text{R} \circ \text{S}$ is an equivalence relation on X

(2) $\text{R} \circ \text{S}$ is symmetric

(3) $\text{R} \circ \text{S}$ is transitive

(4) $\text{R} \circ \text{S} = \text{S} \circ \text{R}$

Functions

1.64 ***Function:*** A relation f from a set A to a set B is called a function from A into B or simply A to B, if for each $a \in A$, there exists unique $b \in B$ such that $(a, b) \in f$. That is $f \subseteq A \times B$ is called a function from A to B, if

(1) Dom $(f) = A$

(2) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$

If f is a function from A to B, then we write $f: A \to B$ is a function and for $(a, b) \in f$, we write $b = f(a)$ and b is called f-image of a and a is called f-preimage of b.

1.65 ***Domain, codomain and range:*** Let $f: A \to B$ be a function. Then A is called domain, B is called codomain and Range of f denoted by $f(A) = \{f(a) \,|\, a \in A\}$. $f(A)$ is also called the image set of A under the function f.

1.66 ***Composition of functions:*** Let $f: A \to B$ and $g: B \to C$ be functions. Then the composition of f with g denoted by $g \circ f$ is defined as $g \circ f: A \to C$ given by

$$(g \circ f)(a) = g(f(a)) \quad \text{for all } a \in A$$

1.67 ***Theorem:*** Let $f: A \to B, g: B \to C$ and $h: C \to D$ be functions. Then

$$(h \circ g) \circ f = h \circ (g \circ f)$$

1.68 ***One-one function or injection:*** A function $f: A \to B$ is called "one-one function" if $f(a_1) \neq f(a_2)$ for any $a_1 \neq a_2$ in A.

1.69 ***Theorem:*** If $f: A \to B$ and $g: B \to C$ be functions. Then the following hold:

(1) If f and g are injections, then so is $g \circ f$.

(2) If $g \circ f$ is an injection, then f is an injection.

1.70 ***Onto function or surjection:*** A function $f: A \to B$ is called "onto function" if the range of f is equal to the codomain B. That is, to each $b \in B$, there exists $a \in A$ such that $f(a) = b$.

1.71 ***Theorem:*** Let $f: A \to B$ and $g: B \to C$ be functions. Then, the following hold:

(1) If f and g are surjections, then so is $g \circ f$.

(2) If $g \circ f$ is a surjection, then g is a surjection.

1.72 ***Bijection or one-one and onto function:*** A function $f: A \to B$ is called "bijection" if f is both an injection and a surjection.

1.73 ***Theorem:*** If $f: A \to B$ and $g: B \to C$ are bijections, then $g \circ f: A \to C$ is a bijection.

1.74 ***Identity function:*** A function $f: A \to A$ is called an identity function if $f(x) = x$ for all $x \in A$ and is denoted by I_A.

1.75 ***Theorem:*** If $f: A \to B$ is a function, then $I_B \circ f = f = f \circ I_A$.

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Identity function is always a bijection.

1.76 ***Theorem:*** Let $f: A \to B$ be a function. Then, f is a bijection if and only if there exists a function $g: B \to A$ such that

$$g \circ f = I_A \quad \text{and} \quad f \circ g = I_B$$

That is

$$g(f(a)) = a \quad \text{for all } a \in A$$

and $$f(g(b)) = b \quad \text{for all } b \in B$$

1.77 ***Inverse of a bijective function:*** Let $f: A \to B$ and $g: B \to A$ be functions such that $g \circ f = I_A$ and $f \circ g = I_B$. Then f and g are bijections. Also g is unique such that $g \circ f = I_A$ and $f \circ g = I_B$. g is called the inverse of f and f is called the inverse of g. The inverse function of f is denoted by f^{-1}.

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If $f: A \to B$ is a bijection, then $f^{-1}: B \to A$ is also a bijection and $f^{-1}(b) = a \Leftrightarrow f(a) = b$ for $b \in B$.

1.78 ***Real-valued function:*** If the range of a function is a subset of the real number set $\mathbb{R}$, then the function is called a real-valued function.

1.79 ***Operations among real-valued functions:*** Let f and g be real-valued functions defined on a set A. Then we define the real-valued functions $f + g$, $-f$, $f - g$ and $f \cdot g$ on the set A as follows:

(1) $(f + g)(a) = f(a) + g(a)$

(2) $(-f)(a) = -f(a)$

(3) $(f - g)(a) = f(a) - g(a)$

(4) $(f \cdot g)(a) = f(a)\, g(a)$

(5) If $g(a) \neq 0$ for all $a \in A$, then

$$\left(\frac{f}{g}\right)(a) = \frac{f(a)}{g(a)}$$

(6) If n is a positive integer, then $f^n(a) = (f(a))^n$.

1.80 ***Integral part and fractional part:*** If x is a real number, then the largest integer less than or equal to x is called the integral part of x and is denoted by $[x]$. $x - [x]$ is called the fractional part of x and will be denoted by $\{x\}$.

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$0 \leq \{x\} < 1$ for any real number x.

1.81 ***Theorem:*** The following hold for any real numbers x and y.

(1) $[x + y] = \begin{cases} [x] + [y] & \text{if } \{x\} + \{y\} < 1 \\ [x] + [y] + 1 & \text{if } \{x\} + \{y\} \geq 1 \end{cases}$

(2) $[x + y] \geq [x] + [y]$ and equality holds if and only if $\{x\} + \{y\} < 1$.

(3) If x or y is an integer, then $[x + y] = [x] + [y]$.

(4) $\left[\dfrac{x}{m}\right] = \left[\dfrac{[x]}{m}\right]$ for any real number x and non-zero-integer m.

(5) If n and k are positive integers and $k > 1$, then

$$\left[\frac{n}{k}\right] + \left[\frac{n+1}{k}\right] \leq \left[\frac{2n}{k}\right]$$

1.82 ***Periodic function:*** Let A be a subset of $\mathbb{R}$ and $f: A \to \mathbb{R}$ a function. A positive real number p is called a period of f if $f(x + p) = f(x)$ whenever x and $x + p$ belong to A. A function with a period is called periodic function. Among the periods of f, the least one (if it exists) is called the *least period*.

1.83 ***Step function (greatest integer function):*** Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = [x]$ for all $x \in \mathbb{R}$ where $[x]$ is the largest integer less than or equal to x. This function f is called step function.

1.84 ***Signum function:*** Let $f: \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

is called Signum function and is written as $\text{sign}(x)$.

1.85 ***Increasing and decreasing functions:*** Let A be a subset of $\mathbb{R}$ and $f: A \to \mathbb{R}$ a function. Then, we say that f is an increasing function if $f(x) \leq f(y)$ whenever $x \leq y$. f is said to be decreasing function if $f(x) \geq f(y)$ whenever $x \leq y$.

1.86 ***Symmetric set:*** A subset X of $\mathbb{R}$ is called a symmetric set if $x \in X \Leftrightarrow -x \in X$.

1.87 ***Even function:*** Let X be a symmetric set and $f: X \to \mathbb{R}$ a function. Then f is said to be even function if $f(-x) = f(x)$ for all $x \in X$.

1.88 ***Odd function:*** Let X be a symmetric set and $f: X \to \mathbb{R}$ a function. Then f is said to be odd function if $f(-x) = -f(x)$ for all $x \in X$.

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If f is an odd function on a symmetric set X and 0 belongs to X, then $f(0)$ is necessarily 0.

1.89 ***Theorem:*** Let X be a symmetric set and f, g be functions from X to $\mathbb{R}$. Then, the following hold:

(1) $f \cdot g$ is even if either both f and g are even or both are odd.

(2) $f \cdot g$ is odd if one of them is odd and the other is even.

1.90 ***Theorem:*** Let f be a real valued function defined on a symmetric set X. Then the following hold:

(1) f is even if and only if af is even for any non-zero $a \in \mathbb{R}$.

(2) f is odd if and only if af is odd for any non-zero $a \in \mathbb{R}$.

(3) f is even (odd) if and only if $-f$ is even (odd).

1.91 ***Theorem:*** If f and g are even (odd) functions then so is $f \pm g$.

1.92 ***Theorem:*** Every real-valued function can be uniquely expressed as a sum of an even function and an odd function. The representation is

$$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$$

1.93 ***Number of partions of a finite set:*** Let $P_0 = 1$ and P_n be the number of partions on a finite set with n elements. Then for $n \geq 1$,

$$P_{n+1} = \sum_{r=1}^{n} \binom{n}{r} P_r$$

where $\binom{n}{r}$ is the number of selections of r objects $(0 \leq r \leq n)$ from n distinct objects and this number is equal to

$$\frac{n!}{r!(n-r)!}$$